CHAPTER 3

POLYNOMIALS –GRAPHS

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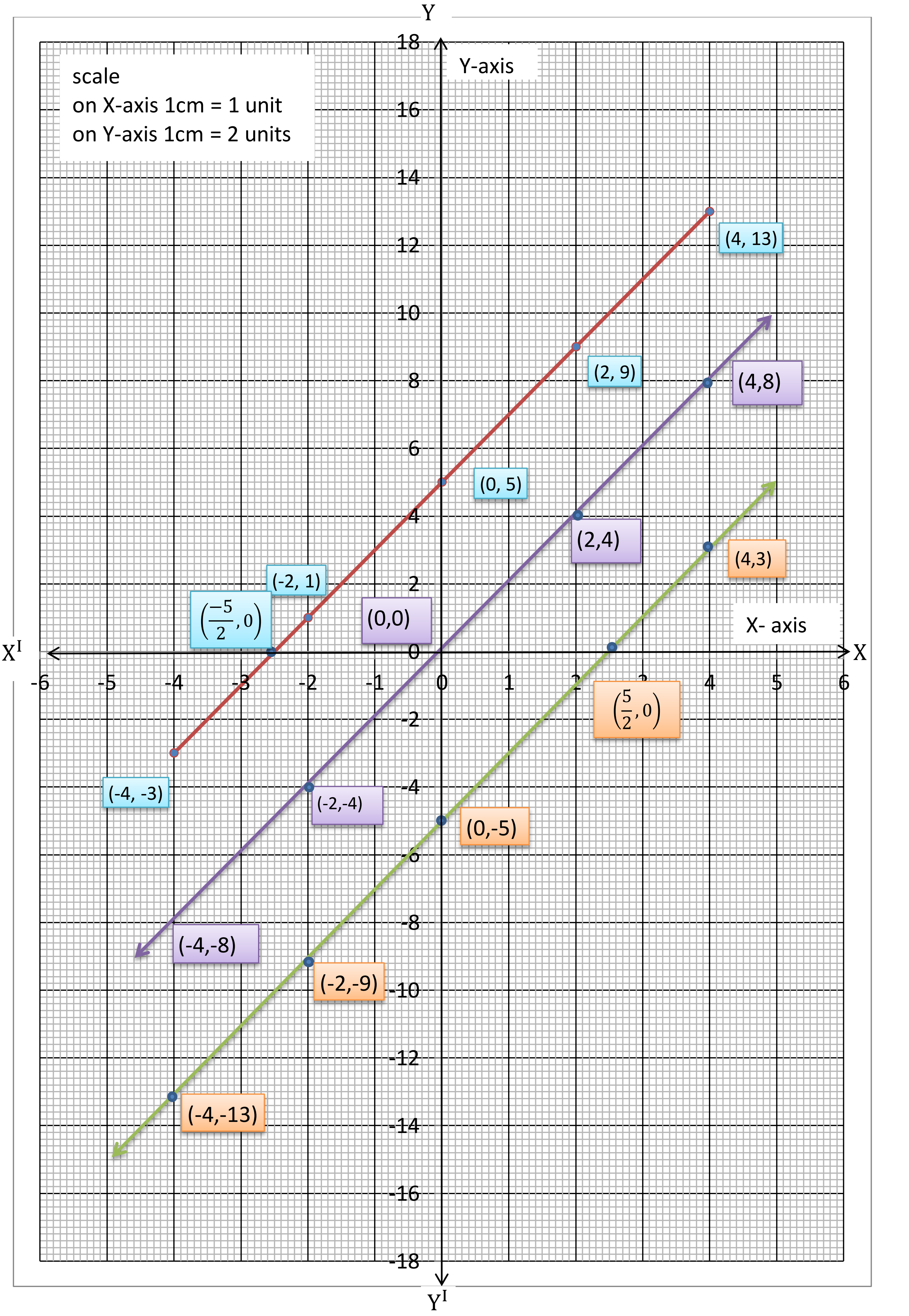
1. The graph of $y = ax + b (a \neq 0)$ is a straight line which intersects the X-axis at exactly one point ,namely $\left(\frac{-b}{a}, 0\right)$.
2. The linear polynomial $ax + b$ $(a \neq 0)$ has exactly one zero $= \frac{-b}{a}$ (the x-coordinate of the point where the graph of $y = ax + b$ intersects the X-axis).
3. The graph of $y = ax^2 + bx + c(a \neq 0)$ either opens upwards like $\cup$ (if $a > 0$) or opens downwards like $\cap$ (if $a < 0$). The shape of these curves are called **parabolas.**
4. The graph of $y = ax^2 + bx + c(a \neq 0)$ intersects X-axis at two points $(a, 0)$ and (b,0) then $\boldsymbol{a}, \boldsymbol{b}$ are the zeroes of the polynomial $ax^2 + bx + c$.
5. The graph of $y = ax^2 + bx + c(a \neq 0)$ touches X-axis one point at $(a, 0)$ then $'\boldsymbol{a}'$ is only one zero of of the polynomial $ax^2 + bx + c$.
6. The graph of $y = ax^2 + bx + c(a \neq 0)$ does not intersects X-axis then the polynomial $ax^2 + bx + c$ has no zeroes .
7. Every linear polynomial have at most one zero.
8. Every quadratic polynomial have at most two zeroes.
9. Every cubic polynomial have at most three zeroes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on X-axis 1cm = 1 unit
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(-4,-8)
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(-4,-13)

Do This

Draw the graph of (i) $y = 2x + 5$, (ii) $y = 2x - 5$, (iii) $y = 2x$ and find the point of intersection on X-axis. Is the x-coordinate of these points also the zeroes of the polynomial?

(i). $y = 2x + 5 = p(x)$

x	-4	-2	0	2	4
$2x$	-8	-4	0	4	8
5	5	5	5	5	5
$y = 2x + 5$	-3	1	5	9	13
(x, y)	$(-4, -3)$	$(-2,1)$	$(0,5)$	$(2,9)$	$(4,13)$

The graph of $y = 2x + 5$ intersects X- axis at $\left(\frac{-5}{2}, 0\right)$. $x - cordinate\ of\ the\ point = \frac{-5}{2}$

$$p\left(\frac{-5}{2}\right) = 2 \times \left(\frac{-5}{2}\right) + 5 = -5 + 5 = 0$$

The zero of the polynomial $y = 2x + 5$ is $\frac{-5}{2}$.

(ii). $y = 2x - 5 = p(x)$

x	-4	-2	0	2	4
$2x$	-8	-4	0	4	8
-5	-5	-5	-5	-5	-5
$y = 2x + 5$	-13	-9	-5	-1	3
(x, y)	$(-4, -13)$	$(-2, -9)$	$(0, -5)$	$(2, -1)$	$(4,3)$

The graph of $y = 2x - 5$ intersects X- axis at $\left(\frac{5}{2}, 0\right)$. $x - cordinate\ of\ the\ point = \frac{5}{2}$

$$p\left(\frac{5}{2}\right) = 2 \times \left(\frac{5}{2}\right) - 5 = 5 - 5 = 0$$

The zero of the polynomial $y = 2x - 5$ is $\frac{5}{2}$.

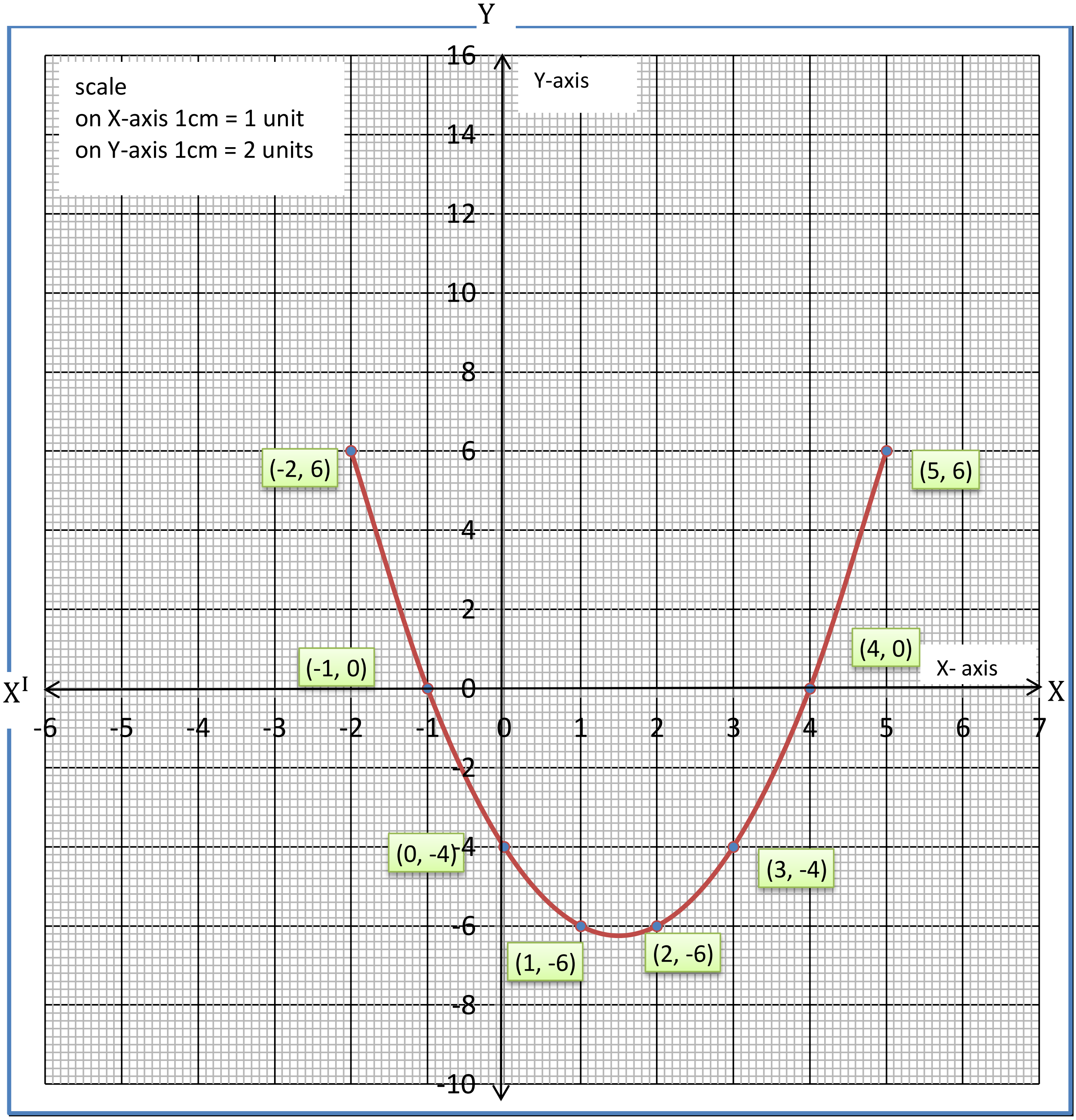
(iii). $y = 2x = p(x)$

x	-4	-2	0	2	4
$y = 2x$	-8	-4	0	4	8
(x, y)	$(-4, -8)$	$(-2, -4)$	$(0,0)$	$(2,4)$	$(4,8)$

The graph of $y = 2x$ intersects X- axis at $(0,0)$. $x - cordinate\ of\ the\ point = 0$

$p(0) = 2 \times (0) = 0$

The zero of the polynomial $y = 2x$ is 0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Example: Draw the graph of polynomial $P(\mathrm{x}) = x^2 - 3x - 4$ and find the zeroes . Justify the answers.

Sol: $P(\mathrm{x}) = x^2 - 3x - 4 = y$

x	-2	-1	0	1	2	3	4	5
x^2	4	1	0	1	4	9	16	25
$-3x$	6	3	0	-3	-6	-9	-12	-15
-4	-4	-4	-4	-4	-4	-4	-4	-4
$y = x^2 - 3x - 4$	-6	0	-4	-6	-6	-4	0	6
(x, y)	$(-2,6)$	$(-1,0)$	$(0,-4)$	$(1,-6)$	$(2,-6)$	$(3,-4)$	$(4,0)$	$(5,6)$

Graph of $y = x^2 - 3x - 4$ intersects X-axis at (-1,0) and (4,0).

Zeroes of the given polynomial $P(\mathrm{x}) = x^2 - 3x - 4$ are -1 *and* 4.

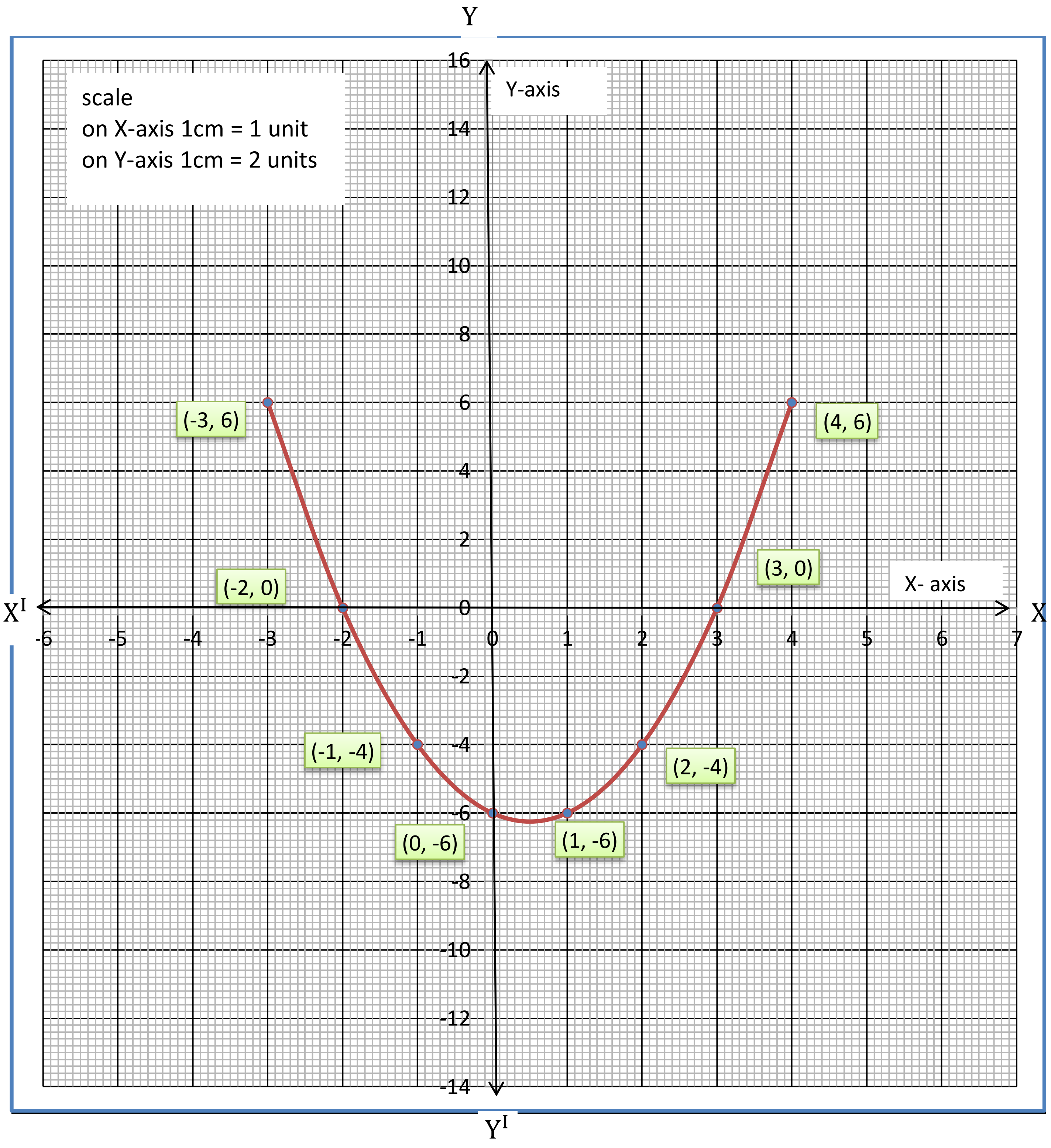
Justification: $P(\mathrm{x}) = x^2 - 3x - 4$

$$P(-1) = (-1)^2 - 3(-1) - 4$$
$$= 1 + 3 - 4$$
$$= 4 - 4$$
$$= 0$$

$$P(4) = 4^2 - 3 \times 4 - 4$$
$$= 16 - 12 - 4$$
$$= 16 - 16$$
$$= 0$$

$$P(-1) = 0 \;\; and \;\; P(4) = 0$$

$$\therefore\ -1\ and\ 4\ are\ the\ zeroes\ of\ the\ polynomial\ P(x) = x^2 - 3x - 4$$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TRY THIS:(**Page-53**):

(i) Draw the graph of $y = x^2 - x - 6$ and find zeroes in each case. What do you notice?

Sol: $p(\text{x}) = x^2 - x - 6 = y$

x	-3	-2	-1	-0	1	2	3	4
x^2	9	4	1	0	1	4	9	16
$-x$	3	2	1	0	-1	-2	-3	-4
-6	-6	-6	-6	-6	-6	-6	-6	-6
$y = x^2 - x - 6$	6	0	-4	-6	-6	-4	0	6
(x, y)	$(-3,6)$	$(-2,0)$	$(-1,-4)$	$(0,-6)$	$(1,-6)$	$(2,-4)$	$(3,0)$	$(4,6)$

Graph of $y = x^2 - x - 6$ intersects X-axis at (-2,0) and (3,0).

Zeroes of the polynomial $p(\text{x}) = x^2 - x - 6$ are -2 *and* 3

Justification: $P(\text{x}) = x^2 - x - 6$

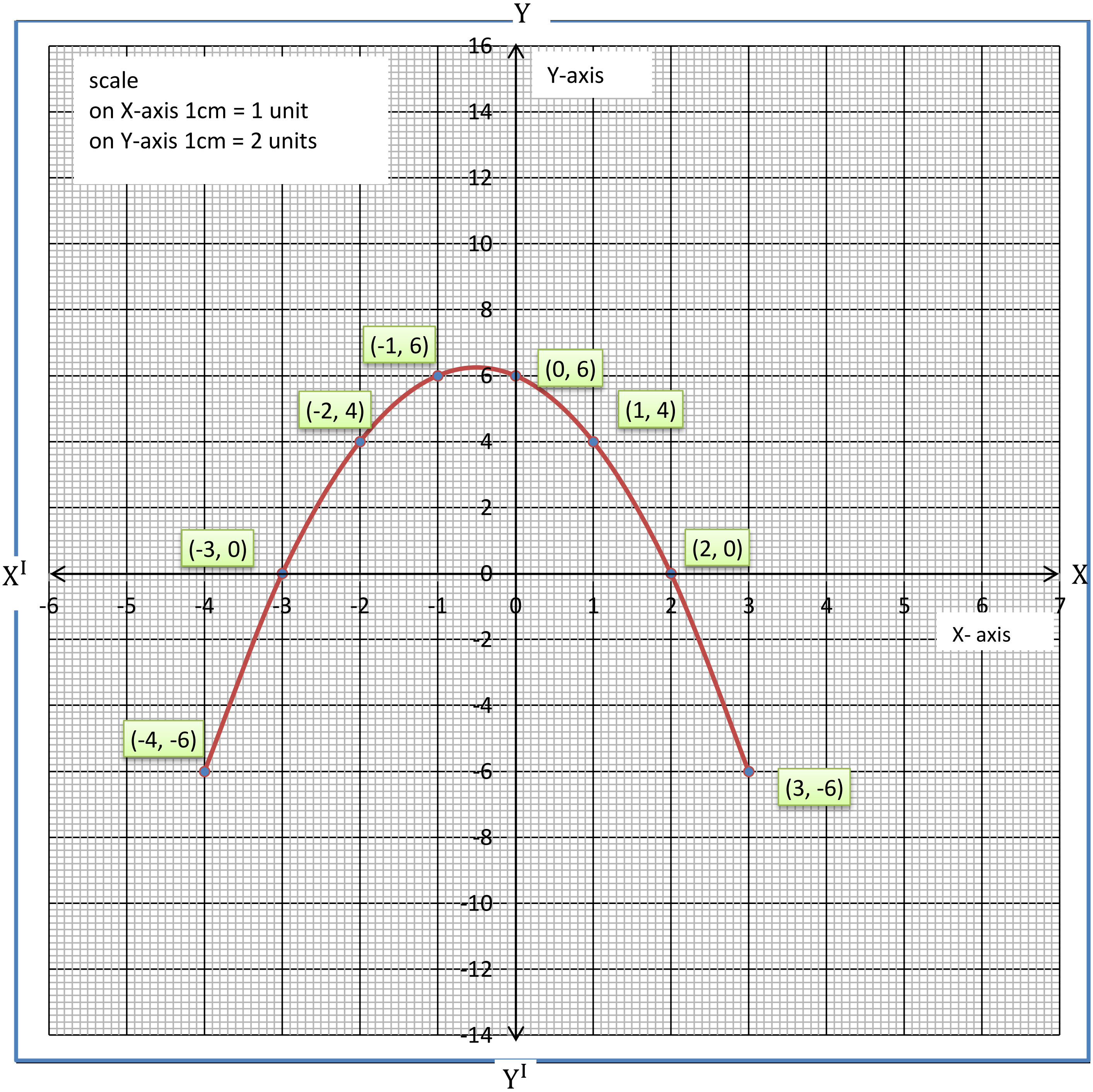
$$P(-2) = (-2)^2 - (-2) - 6$$
$$= 4 + 2 - 6$$
$$= 6 - 6$$
$$= 0$$

$$P(3) = 3^2 - 3 - 6$$
$$= 9 - 3 - 6$$
$$= 9 - 9$$
$$= 0$$

$$P(-2) = 0 \ \ and \ \ P(3) = 0$$

$$\therefore \ -2 \ and \ 3 \ are \ zeroes \ of \ polynomial \ P(x) = x^2 - x - 6$$

We notice that in polynomial $p(x) = ax^2 + bx + c$ if $a > 0$ then the graph (parabola) opens upwards like $\cup$.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-6 -5 -4 -3 -2 -1 0 1 2 3 4 5 6 7
X- axis
Y^{I}

(ii) Draw the graph of $y = 6 - x - x^2$ and find zeroes in each case .What do you notice?

Sol: $p(\mathrm{x}) = 6 - x - x^2 = -x^2 - x + 6 = y$

x	-4	-3	-2	-1	-0	1	2	3
$-x^2$	-16	-9	-4	-1	0	-1	-4	-9
$-x$	4	3	2	1	0	-1	-2	-3
6	6	6	6	6	6	6	6	6
$y = 6 - x - x^2$	-6	0	4	6	6	4	0	-6
(x, y)	$(-4,-6)$	$(-3,0)$	$(-2,4)$	$(-1,6)$	$(0,6)$	$(1,4)$	$(2,0)$	$(3,-6)$

Graph of $y = 6 - x - x^2$ intersects X-axis at (-3,0) and (2,0).

Zeroes of the polynomial $p(\mathrm{x}) = 6 - x - x^2$ are -3 *and* 2

Justification: $P(\mathrm{x}) = 6 - x - x^2$

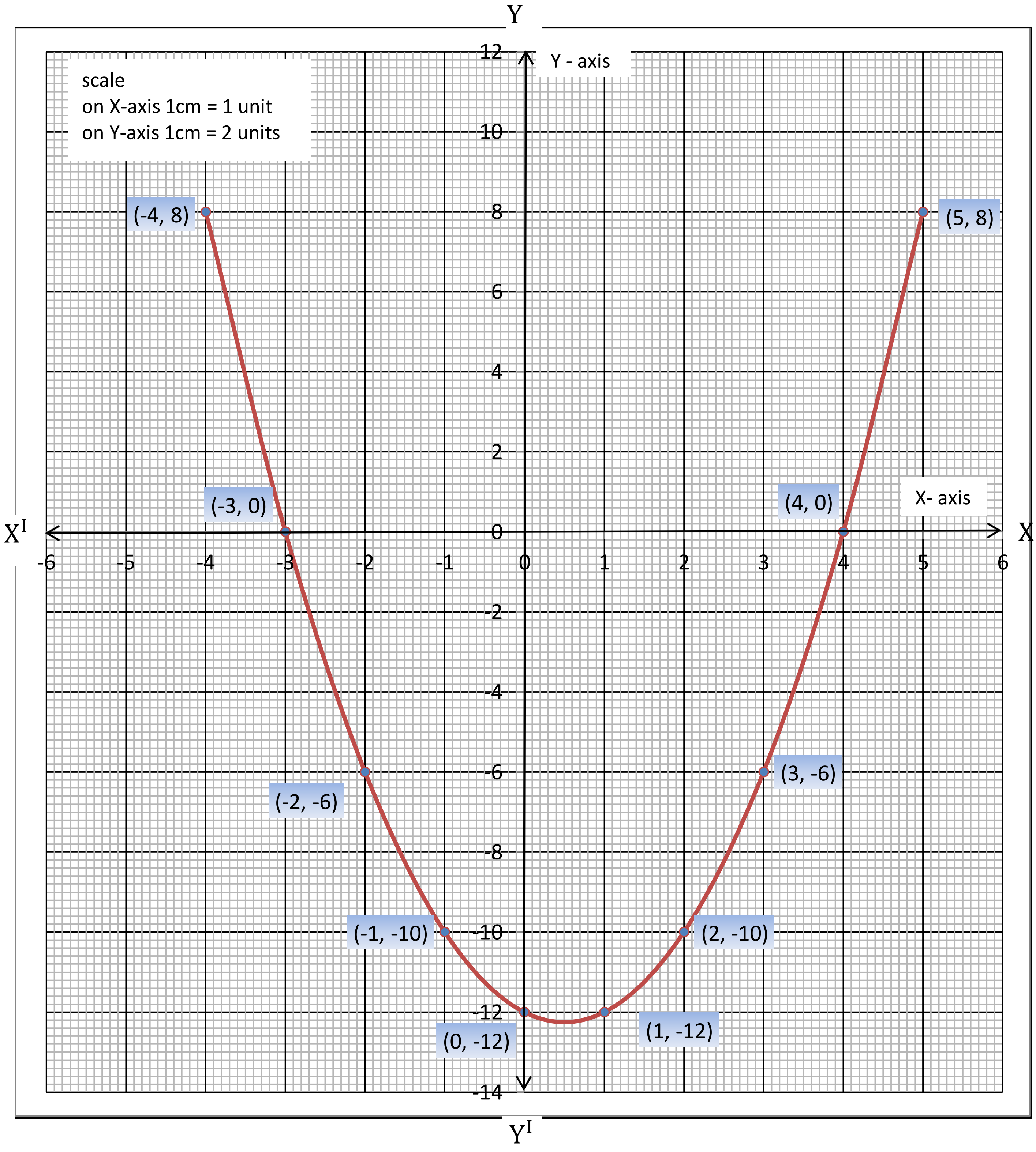
$$P(-3) = 6 - (-3) - (-3)^2$$
$$= 6 + 3 - 9$$
$$= 9 - 9$$
$$= 0$$

$$P(2) = 6 - 2 - 2^2$$
$$= 6 - 2 - 4$$
$$= 6 - 6$$
$$= 0$$

$$P(-3) = 0 \ \ and\ P(2) = 0$$

$$\therefore\ -3\ and\ 2\ are\ the\ zeroes\ of\ the\ polynomial\ P(x) = 6 - x - x^2$$

We notice that in polynomial $p(x) = ax^2 + bx + c$ if $a < 0$ then the graph (parabola) opens downwards like $\cap$.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on Y-axis 1cm = 2 units
(-4, 8)
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X- axis
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(-2, -6)
(3, -6)
(-1, -10)
(2, -10)
(0, -12)
(1, -12)
Y^I

EXERCISE – 3.2

3. (i) Draw the graph of polynomial $p(x) = x^2 - x - 12$and find the zeroes in each case . Justify the answers .

Sol: P$(x) = x^2 - x - 12 = y$

x	-4	-3	-2	-1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
$-x$	4	3	2	1	0	-1	-2	-3	-4
-12	-12	-12	-12	-12	-12	-12	-12	-12	-12
$y = x^2 - x - 12$	0	0	-6	-10	-12	-12	-10	-6	0
(x, y)	(-4,8)	(-3,0)	(-2,-6)	(-1,-10)	(0,-12)	(1,-12)	(2,-10)	(3,-6)	(4,0)

Graph of $y = x^2 - x - 12$ intersects the X- axis at (-3,0) and (4,0).

The zeroes of the polynomial P$(x) = x^2 - x - 12$ are **-3** and **4.**

Justification:- P$(x) = x^2 - x - 12$

P(-3) $= (-3)^2 - (-3) - 12$

$= 9 + 3 - 12$

$= 12 - 12$

$= 0$

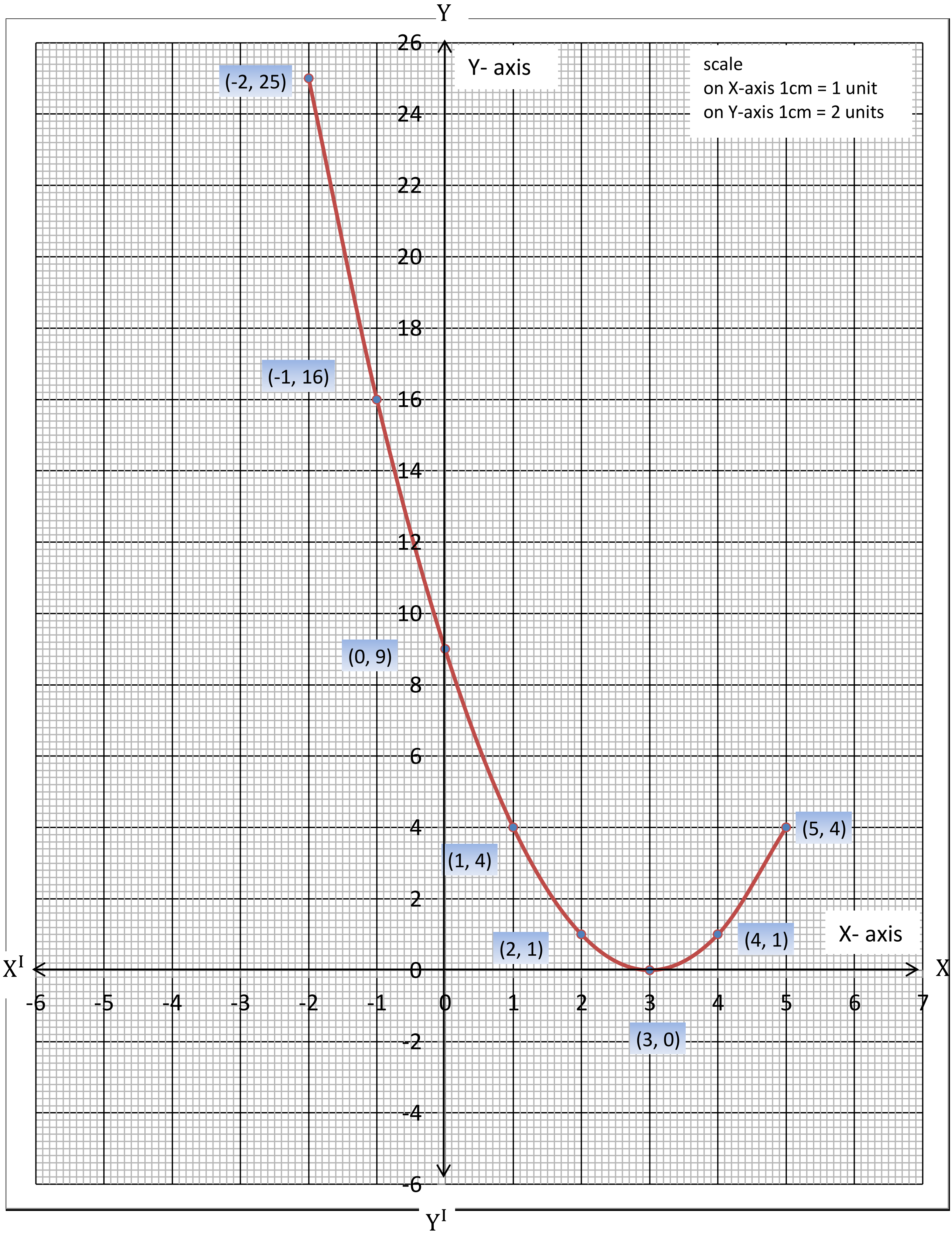
P(4) $= (4)^2 - 4 - 12$

$=16-16$

$= 0$

P(-3) =0 and P(4) = 0

∴ -3 and 4 are the zeroes of the polynomial P$(x) = x^2 - x - 12$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(1, 4)
(2, 1)
(3, 0)
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(ii) Draw the graph of polynomial $p(x) = x^2 - 6x + 9$and find the zeroes in each case . Justify the answers .

Sol: $P(\mathrm{x}) = x^2 - 6x + 9 = y$

x	-2	-1	0	1	2	3	4	5
x^2	4	1	0	1	4	9	16	25
$-6x$	12	6	0	-6	-12	-18	-24	-30
9	9	9	9	9	9	9	9	9
$y = x^2 - 6x + 9$	25	16	9	4	1	0	1	4
(x, y)	(-2,25)	(-1,16)	(0,9)	(1,4)	(2,1)	(3,0)	(4,1)	(5,4)

Graph of $y = x^2 - 6x + 9$ touches X-axis at (3,0)

Zeroes of the polynomial $P(\mathrm{x}) = x^2 - 6x + 9$ are 3,3.

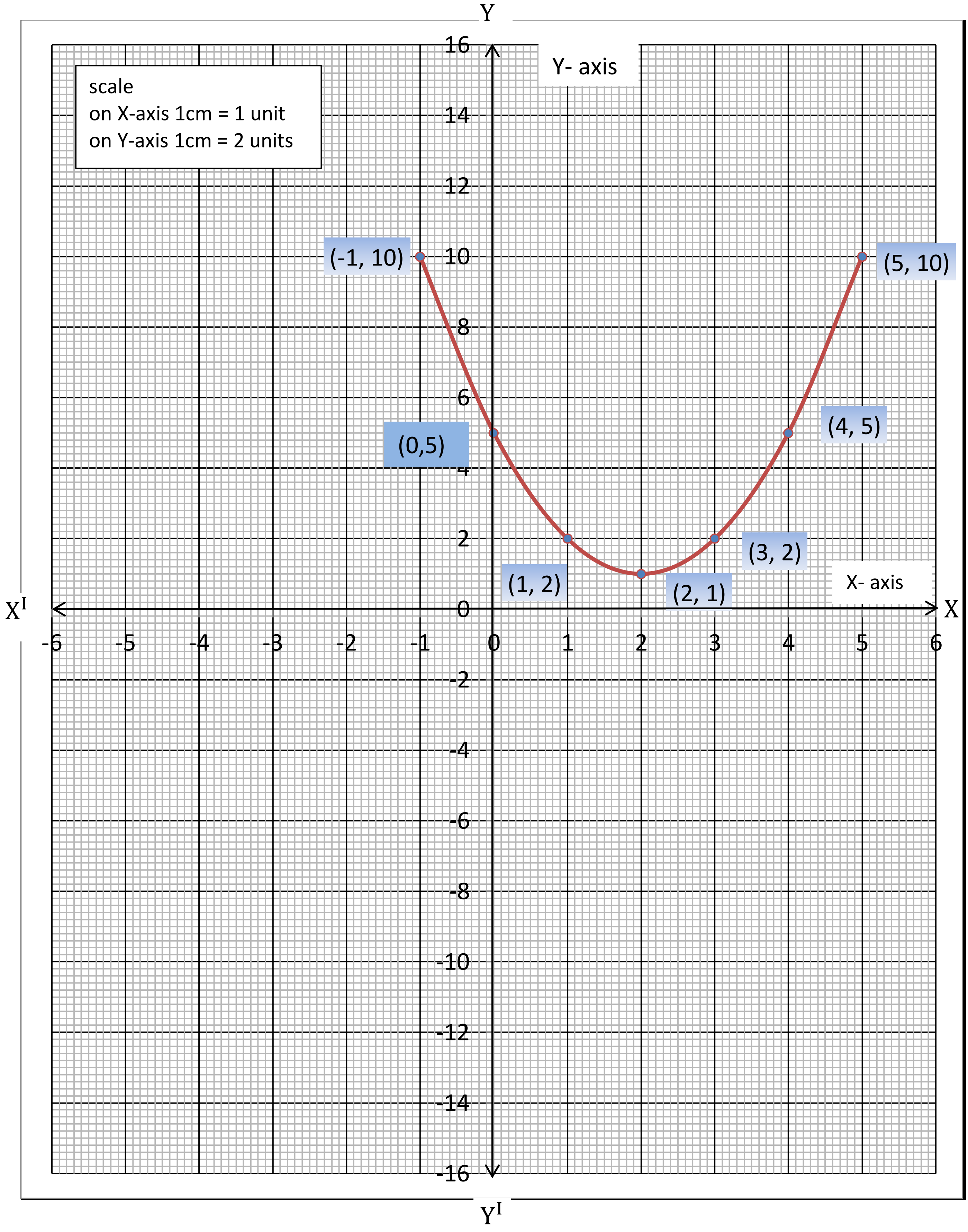
Justification: $P(\mathrm{x}) = x^2 - 6x + 9$

$$P(3) = 3^2 - 6 \times 3 + 9$$

$$= 9 - 18 + 9$$

$$= 18 - 18 = 0$$

$\therefore 3\ \textit{is the zero of the polynomial}\ P(x) = x^2 - 6x + 9$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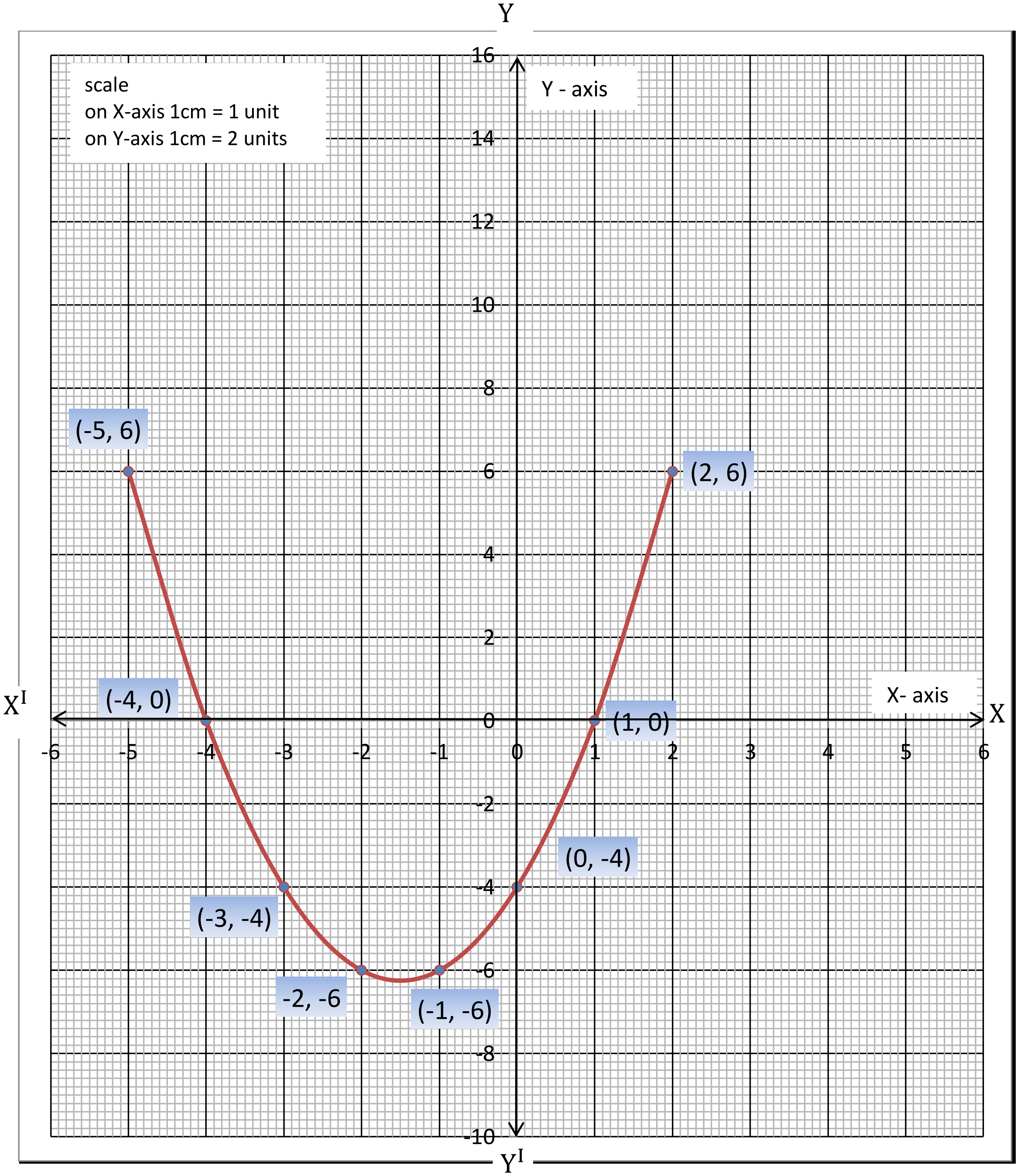
(iii) Draw the graph of polynomial $p(x) = x^2 - 4x + 5$and find the zeroes . Justify the answers

Sol: $\mathrm{P}(x) = x^2 - 4x + 5 = \mathrm{y}$

x	-2	-1	0	1	2	3	4
x^2	4	1	0	1	4	9	16
$-4x$	8	4	0	-4	-8	-12	-16
5	5	5	5	5	5	5	5
$y = x^2 - 4x + 5$	17	10	5	2	1	2	5
(x, y)	(-2,17)	(-1,10)	(0,5)	(1,2)	(2,1)	(3,3)	(4,5)

Graph of $y = x^2 - 4x + 5$ does not intersects the X- axis .

So there are no real zeroes for the given polynomial $\mathrm{P}(x) = x^2 - 4x + 5$.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(iv) Draw the graph of polynomial $p(x) = x^2 + 3x - 4$ and find the zeroes . Justify the answers

Sol: $P(x) = x^2 + 3x - 4 = y$

x	-5	-4	-3	-2	-1	0	1	2
x^2	25	16	9	4	1	0	1	4
$3x$	-15	-12	-9	-6	-3	0	3	6
-4	-4	-4	-4	-4	-4	-4	-4	-4
$y = x^2 + 3x - 4$	6	0	-4	-6	-6	-4	0	6
(x, y)	(-5,6)	(-4,0)	(-3,-4)	(-2,-6)	(-1,-6)	(0,-4)	(1,0)	(2,6)

Graph of $y = x^2 + 3x - 4$ intersects the X- axis at (-4,0) and (1,0).
So the zeroes of the polynomial $P(x) = x^2 + 3x - 4$are **-4** and **1.**

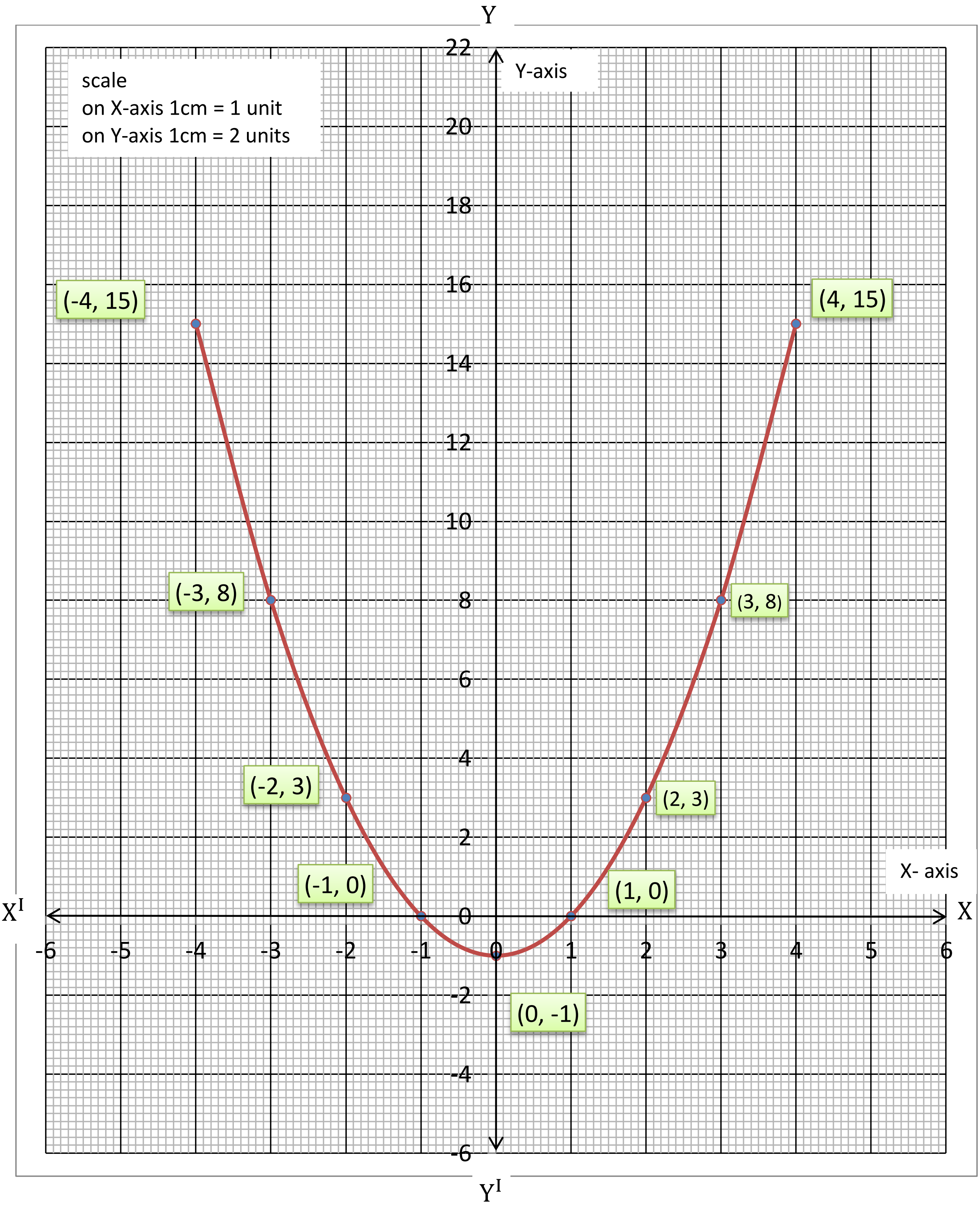
Justification:- $P(x) = x^2 + 3x - 4$

$P(-4) = (-4)^2 + 3(-4) - 4$
$= 16 - 12 - 4$
$= 16-16$
$= 0$

$P(1) = (1)^2 + 3 \times 1 - 4$
$=1+3 - 4$
$= 4 - 4$
$= 0$

P(-4) =0 and P(1) = 0

$\therefore\ -4\ and\ 1\ are\ the\ zeroes\ of\ the\ polynomial\ P(x) = x^2 + 3x - 4$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(v) Draw the graph of polynomial P(x) $= x^2 - 1$ and find the zeroes . Justify the answers

Sol: P(x) $= x^2 - 1 = y$

x	-4	-3	-2	-1	0	1	2	3	4
x^2	16	9	4	1	0	1	4	9	16
-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
$y = x^2 - 1$	15	8	3	0	-1	0	3	8	15
(x, y)	(-4,15)	(-3,8)	(-2,3)	(-1,0)	(0,-1)	(1,0)	(2,3)	(3,8)	(4,15)

The graph of $y = x^2 - 1$ intersects the X- axis at (-1,0) and (1,0).

So the zeroes of the polynomial P(x) $= x^2 - 1$are **-1** and **1.**

Justification: P(x) $= x^2 - 1$

$P(1) = 1^2 - 1$ $\quad$ $P(-1) = (-1)^2 - 1$

$= 1 - 1$ $\quad$ $= 1 - 1$

$= 0$ $\quad$ $= 0$

P(1) =0 and P(-1) = 0

∴ 1 and -1 are the zeroes of the polynomial $P(x) = x^2 - 1$

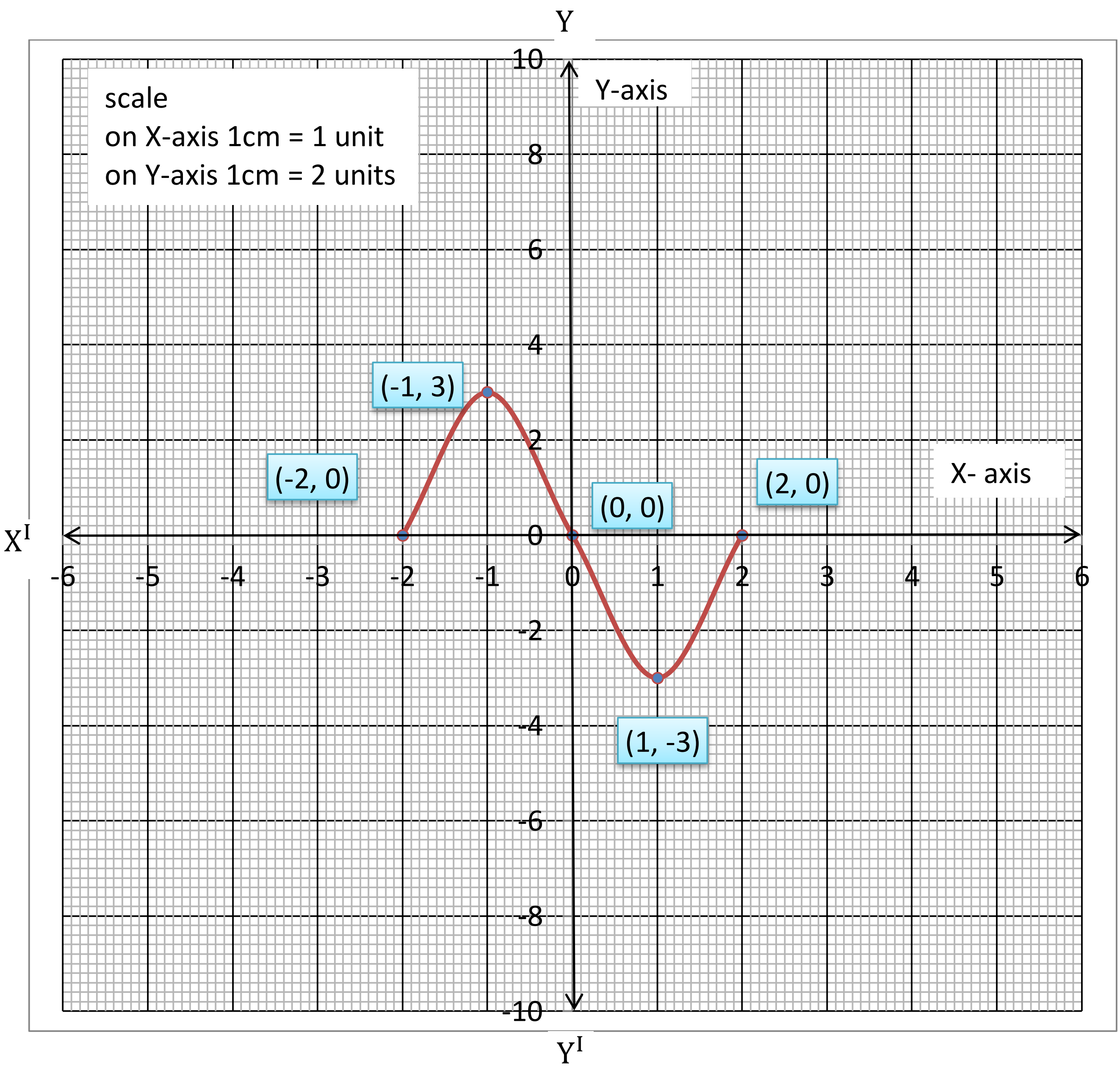
1. Draw the graph of cubic polynomial $p(x) = x^3 - 4x$ and find the zeroes of the polynomial.

Sol: $p(x) = x^3 - 4x = y$

x	-2	-1	0	1	2
x^3	-8	-1	0	1	8
$-4x$	8	4	0	-4	-8
$y = x^3 - 4x$	0	3	0	-3	0
(x, y)	$(-2,0)$	$(-1,3)$	$(0,0)$	$(1,-3)$	$(2,0)$

The graph of $y = x^3 - 4x$ intersects the x-axis at $(-2,0)$, $(0,0)$ and $(2,0)$

The zeroes of the cubic polynomial $p(x) = x^3 - 4x$ are $-2, 0$ *and* 2



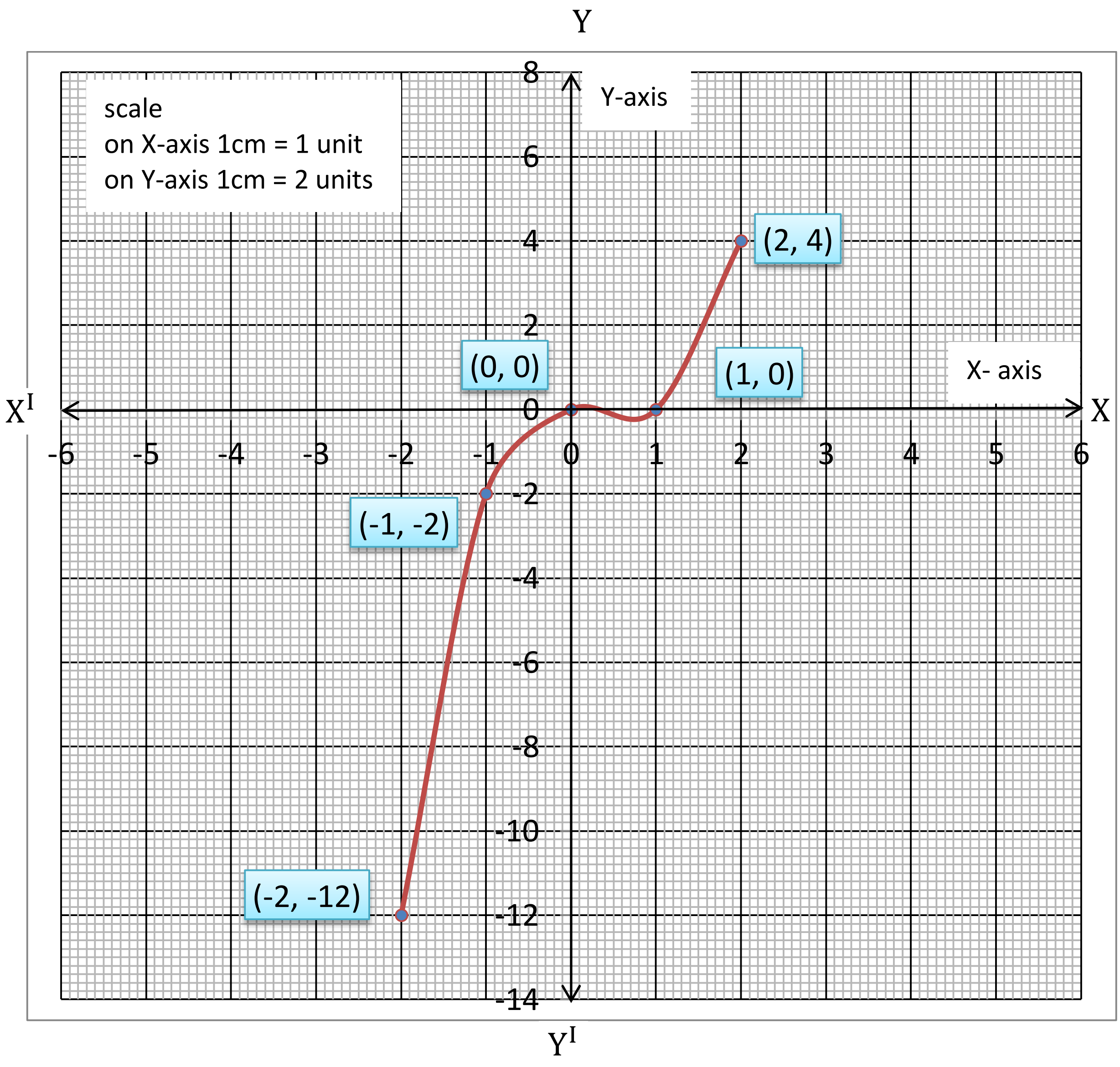
2. .Draw the graph of cubic polynomial $p(x) = x^3$ and find the zeroes of the polynomial.

Sol: $p(x) = x^3 = y$

x	-2	-1	0	1	2
$y = x^3$	-8	-1	0	1	8
(x, y)	$(-2, -8)$	$(-1, -1)$	$(0,0)$	$(1,1)$	$(2,8)$

The graph of $y = x^3$ intersects the x-axis at (0,0)

The zeroes of the cubic polynomial $p(x) = x^3$is 0 .



3. Draw the graph of cubic polynomial $p(x) = x^3 - x^2$ and find the zeroes of the polynomial.

Sol: $p(x) = x^3 - 4x = y$

x	-2	-1	0	1	2
x^3	-8	-1	0	1	8
$-x^2$	-4	-1	0	-1	-4
$y = x^3 - x^2$	-12	-2	0	0	4
(x, y)	$(-2, -12)$	$(-1, -2)$	$(0,0)$	$(1,0)$	$(2,4)$

The graph of $y = x^3 - x^2$ intersects the x-axis at (0,0) and (1,0)

The zeroes of the cubic polynomial $p(x) = x^3 - 4x$ are 0 *and* 1.